Visibility Graphs of Staircase Polygons

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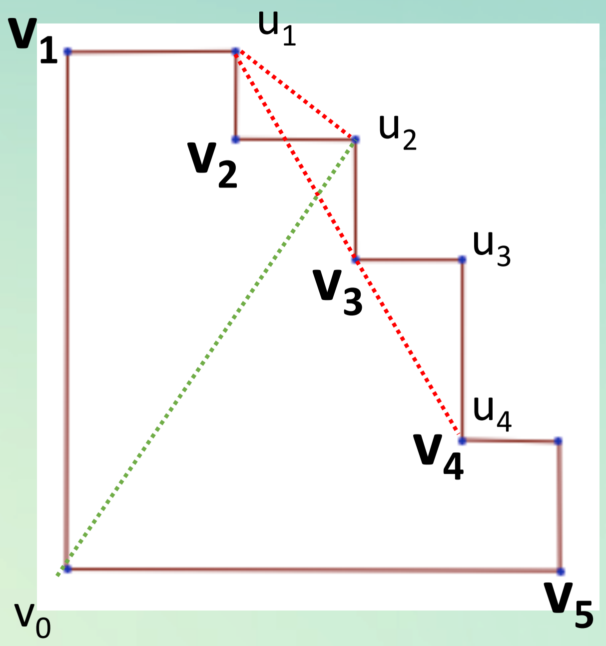
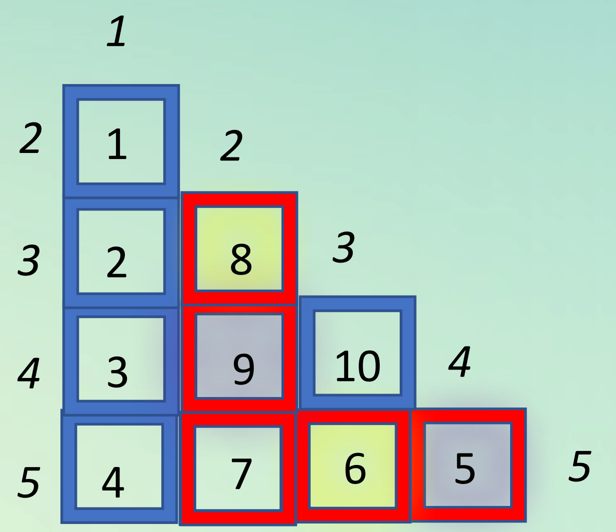
Let me remind you…

- We consider a simple non-degenerate collection of points in the plane that produces a ***polygon***
- In particular, we look at ***staircase polygon paths***
- Two vertices of a polygon are called ***internally visible*** if the closed line segment between them is either an edge of the polygon or lies entirely in the interior of the polygon (Abello et al)
- The ***visibility graph*** of a polygon is a graph whose vertex set is the same as the vertex set of the polygon and whose edges are the straight-line segments between internally visible vertices



Balanced Tableau

- ***Hook of a cell*** is the collection of cells that includes the chosen cell with all the cells above it and all the cells to the right
- ***Mate*** cells with respect to the chosen cell
- A tableau is ***balanced*** if the value of every cell lies in between every pair of mate cells in its hook
- (!) Tableau represents ***slope ranks*** in a staircase path on n vertices



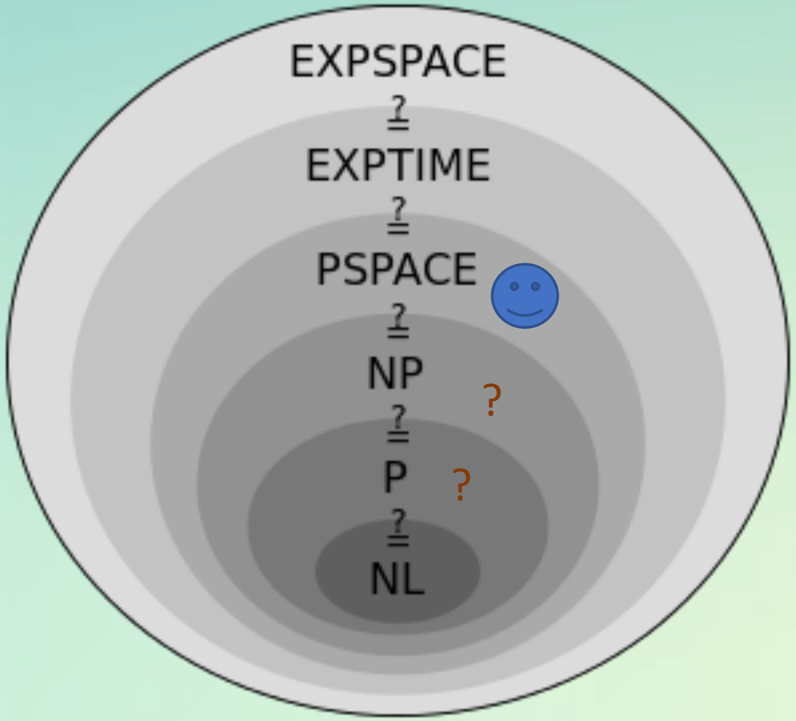
Local Max (Min) Rule

- Apply the rule to obtain the adjacency matrix

1			
2	8		
3	9	10	
4	7	6	5

1			
1	1		
1	1	1	
1	0	0	1

Problem Statement



Problem Statement:
Input: A balanced tableau T_n
Output: Build a staircase polygon s.t. its visibility graph is isomorphic to localmax (T_n)

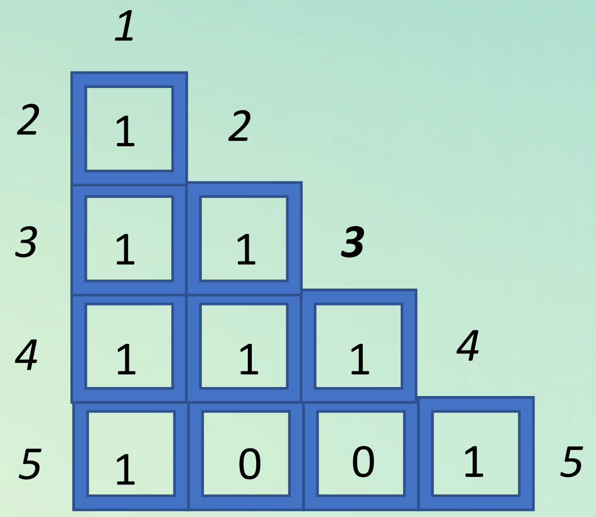
- The problem is known to be PSPACE
- We also want to know whether it is NP or P

What I tried:

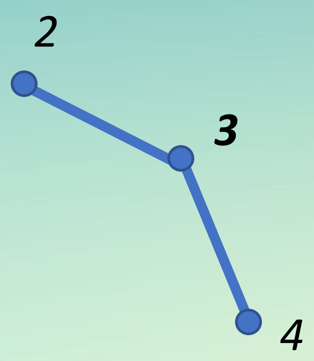
- Random Stuff ☹
- Convex Hull Approach ☹
- Inductive Approach ☹ / ☺
- Visibility Regions Approach 😍

Visibility Regions Approach

- Starts building from the middle
- Takes advantage of unboundedness
- Forms a visibility region to place each new vertex

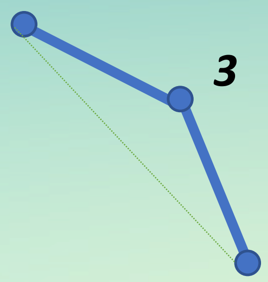


Example



	1	2	**3**	4
2	1			
3	1	1		
4	1	1	1	
5	1	0	0	1

Example



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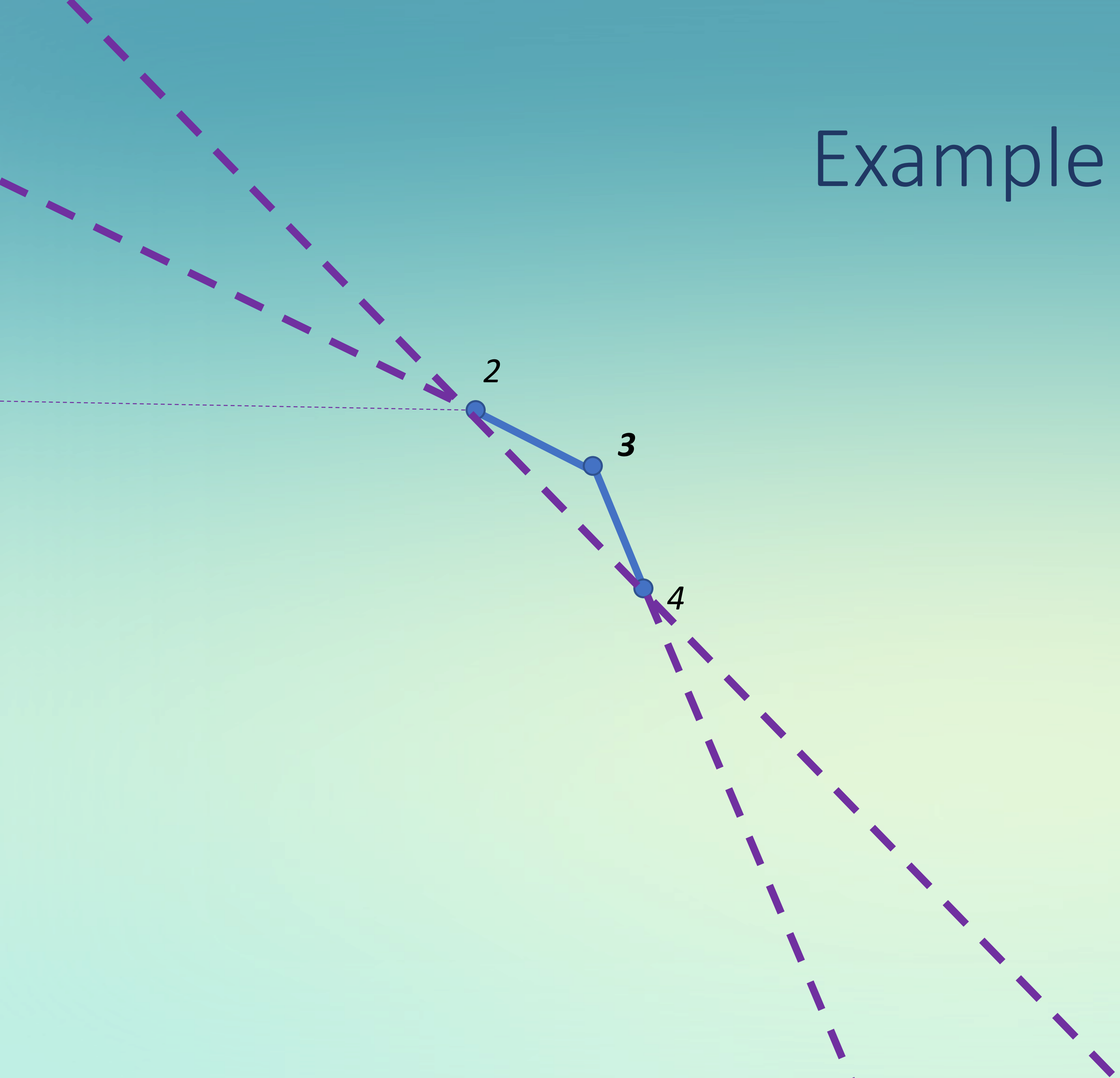

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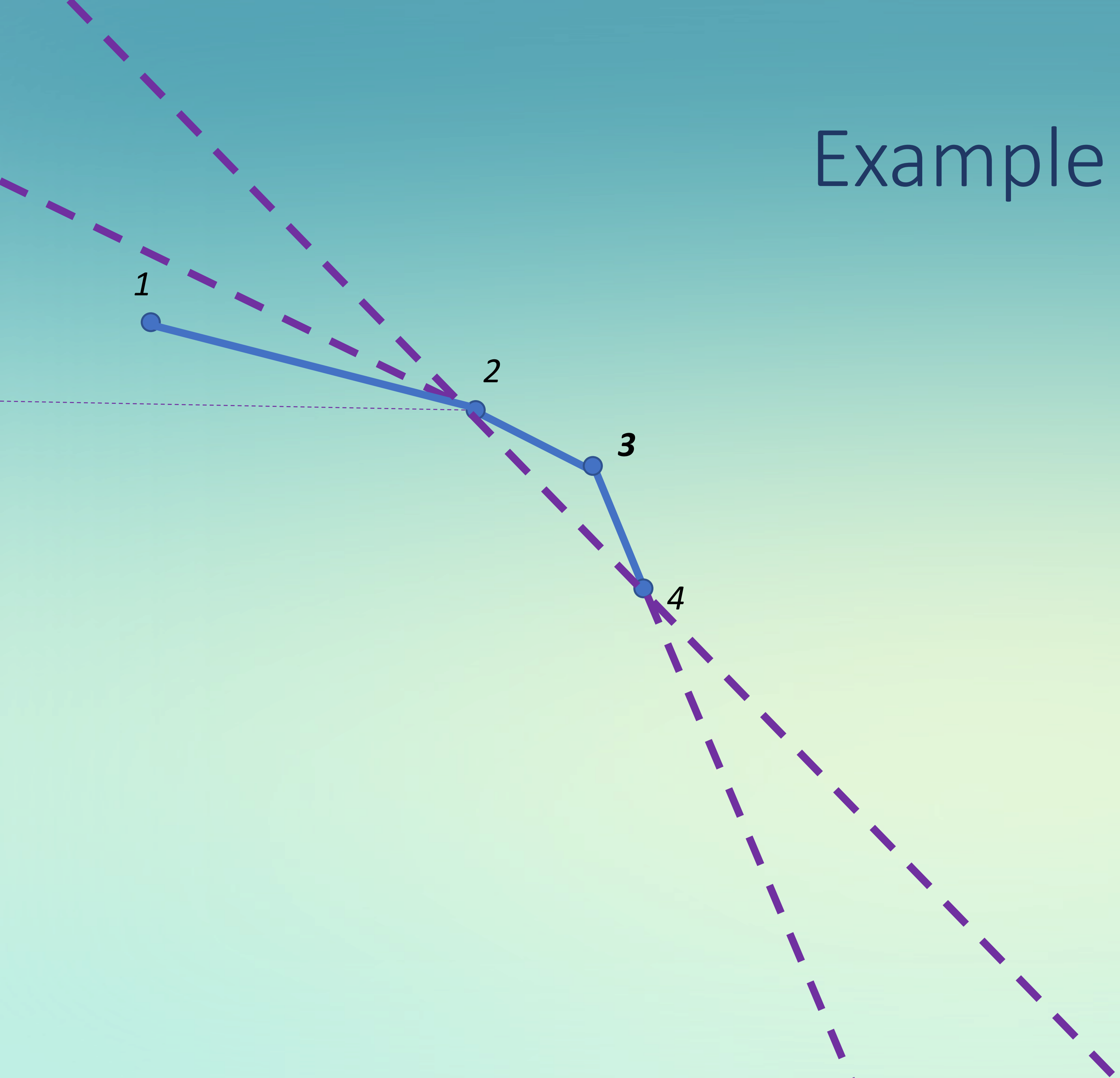
Example



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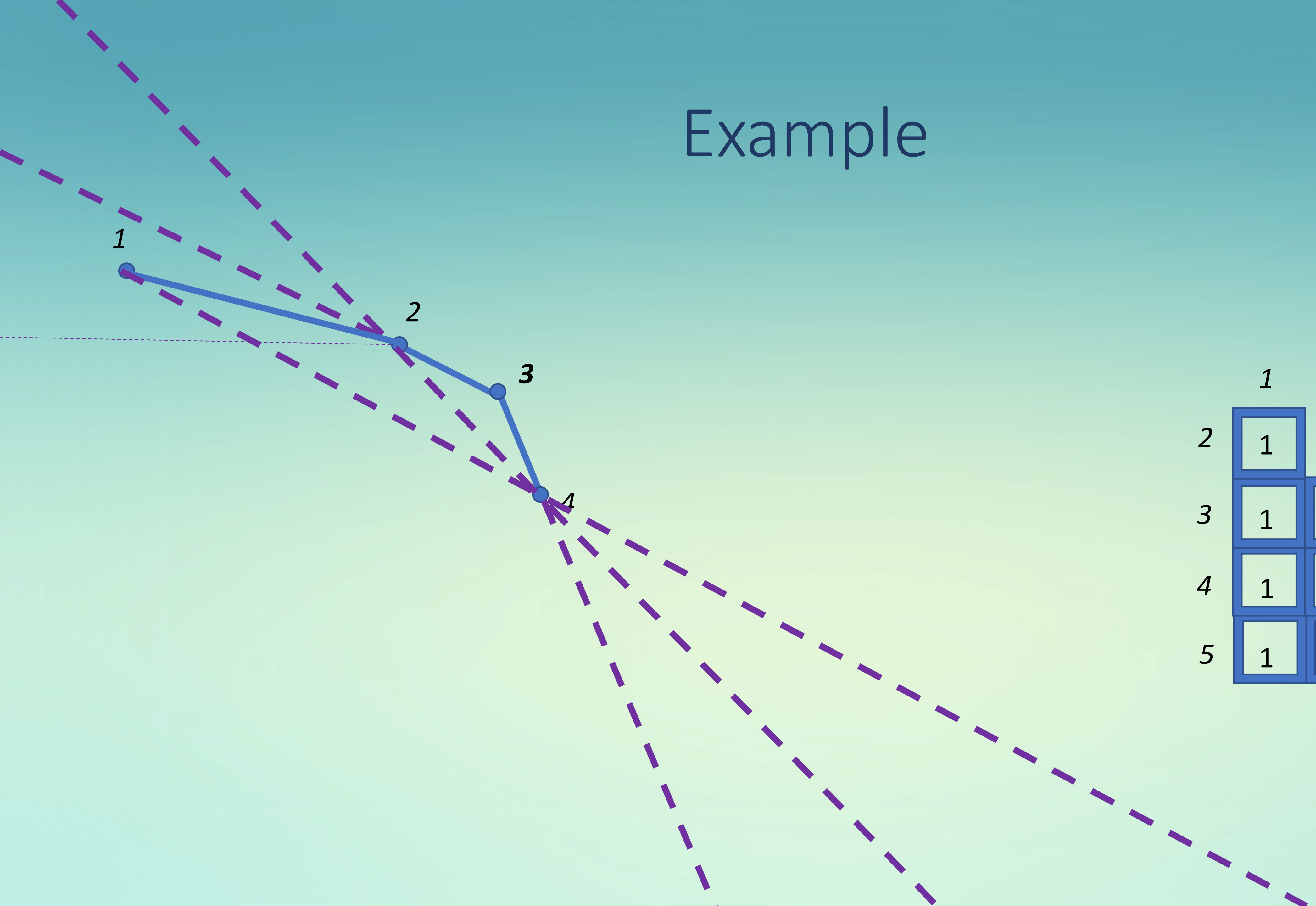
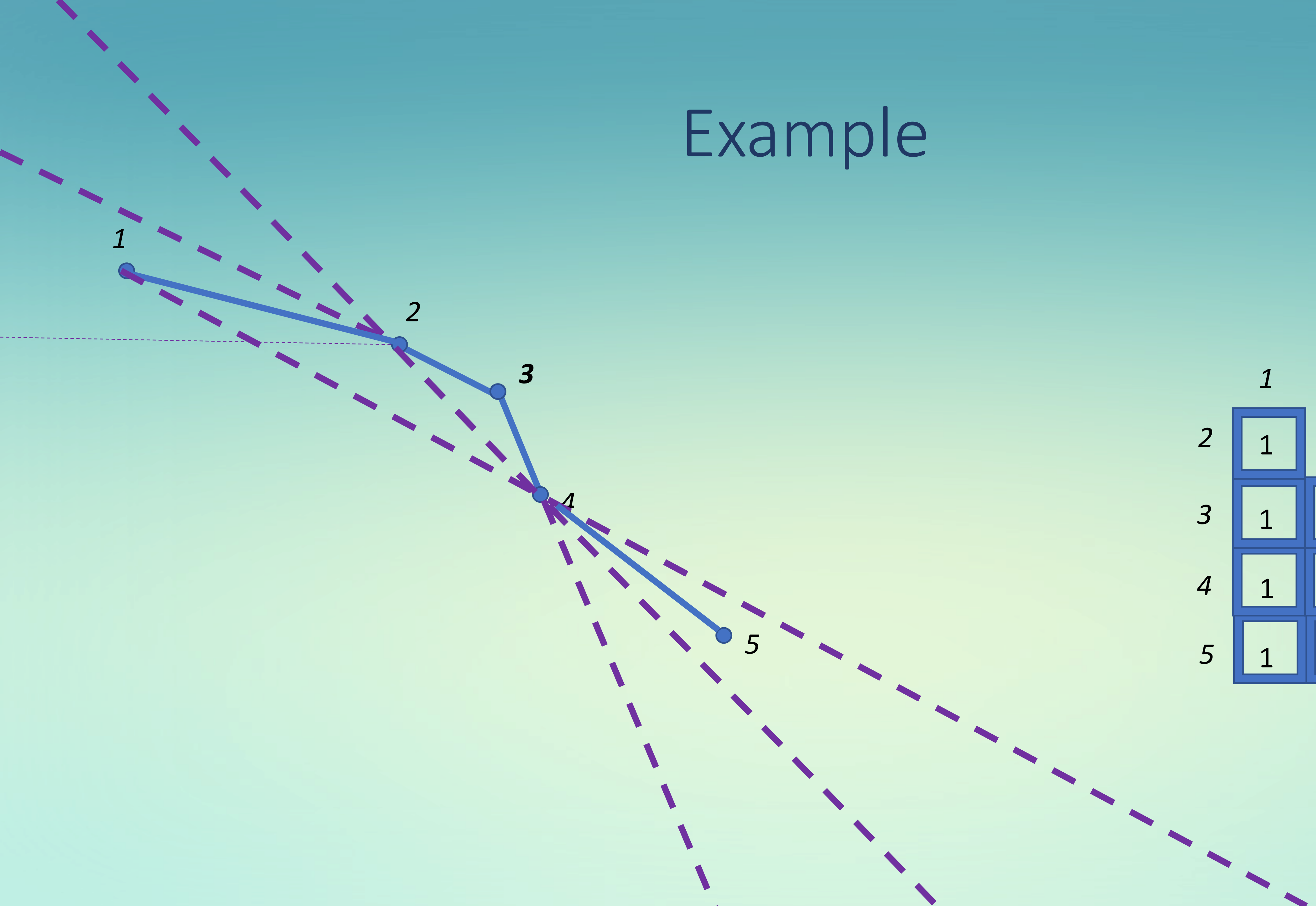
3	1	1		
4	1	1	1	
5	1	0	0	1

Example



	1				
2	1	2			
3	1	1	**3**		
4	1	1	1	4	
5	1	0	0	1	5

Example



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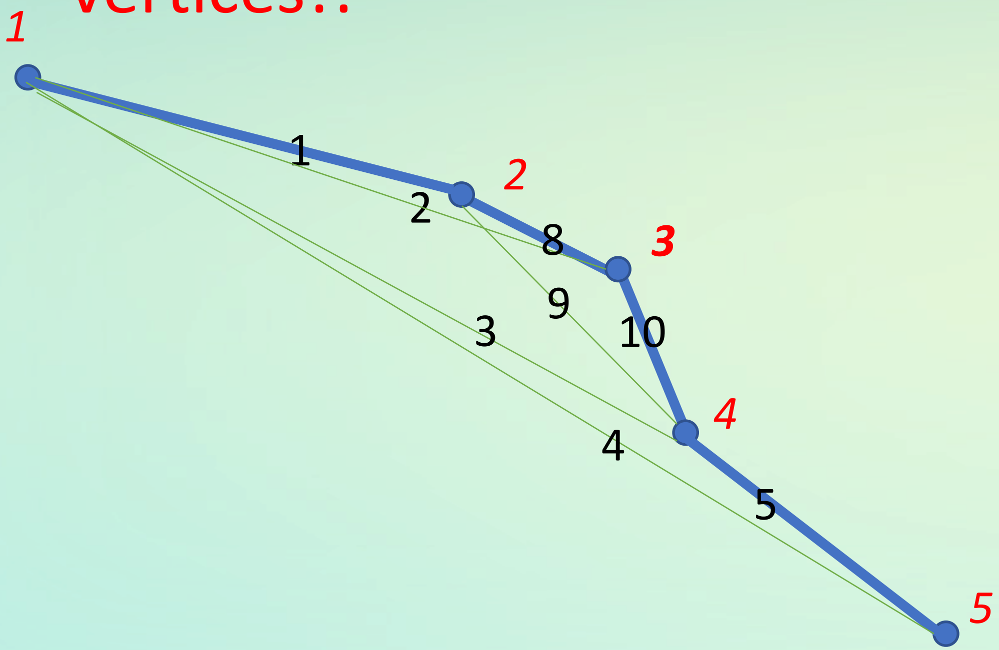
Too good to be true…

- Can visibility regions be empty? Yep.
- Why? Research is hard.
- What makes them empty? Not preserving slope ranks of farthest seen vertices!!



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What I proved:

- Regions are never empty as long as we preserve slope ranks of farthest seen vertices at each stage of construction
  - Concave-concave (convex-convex)
  - Concave-convex (convex-concave)
  - General case

What I proved:

- Regions are never empty as long as we preserve slope ranks of farthest seen vertices at each stage of construction
    - Concave-concave (convex-convex)
    - Concave-convex (convex-concave)
    - General case
- It is always possible to preserve slope ranks of farthest seen vertices

What’s left:

- Determine complexity
- Double check and polish proofs
- Finalize results for publication

Acknowledgements:

- Prof. James Abello
- DIMACS and Prof. Gallos
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Thanks! ☺

- References:

  - [1] Abello et al, *Visibility Graphs of Staircase Polygons and the Weak Bruhat Order, I: from Visibility Graphs to Maximal Chains**. Discrete & Computational Geometry. 1995. 331-358.